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# ANALYTICAL COMPARISON OF RELATIVE HEAT-TRANSFER COEFFICIENTS AND PRESSURE DROPS OF INERT GASES AND THEIR BINARY MIXTURES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1965



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SUMMARY

This analysis was conducted to determine and compare the relative heat-transfer coefficients and pressure drops of the inert gases and their binary mixtures. The required transport properties were determined from kinetic theory correlations, and the relative heat-transfer coefficients and pressure drops were determined from commonly used heat-transfer and pressure-drop equations.

The relative heat-transfer coefficients and pressure drops of the binary mixtures cannot be obtained by common averaging methods from the pure gas values. For a given molecular weight, the heat-transfer coefficients of the binary mixtures are higher than those of the pure component for both laminar and turbulent flow, and the turbulent-flow pressure drops for all the mixtures are comparable to that of the pure component. The nature of the laminar-flow pressure-drop relation precludes any generalization.

The substitution of a higher-molecular-weight binary gas mixture for a pure gas in a gas cycle system can result, in many cases, in more favorable turbomachinery and, at the same time, an improvement in heat-transfer performance.

INTRODUCTION

Brayton cycle systems are currently being given serious consideration for use as space power systems. The transport properties (e.g., viscosity and thermal conductivity) and the thermodynamic properties (e.g., specific heat) of the working fluid (inert gas) strongly affect the design of the Brayton cycle components. The properties of the inert gases are such that an increase in fluid molecular weight causes a decrease in the required number of turbomachinery stages and a potential improvement in performance through an increase in Reynolds number; however, there also occurs a decrease in the heat-transfer coefficient (ref. 1). From the standpoint of the turbomachinery components, therefore, a high molecular weight fluid is desired, while from the standpoint of the heat-transfer components, a lower molecular weight fluid is desired.

The final selection of a fluid for a Brayton cycle system results from compromising these effects.

In view of the interest in Brayton cycle systems, there was a desire to find a working fluid with improved heat-transfer properties relative to turbomachinery properties. The use of inert gas mixtures as the working fluid was suggested in references 2 and 3. The larger molecule would contribute the favorable molecular weight while the smaller molecule would contribute favorable heat-transfer properties. The use of binary mixtures of the inert gases would be sufficient to show this effect. The thermodynamic properties of these mixtures can be readily calculated by appropriate averaging of the pure gas properties. The transport properties, however, require more complex considerations. Experimentally determined transport properties of some binary mixtures are available at room temperatures, but not at the temperatures of interest for Brayton cycle systems. There are available, however, estimation techniques, based on the kinetic theory of gases, that yield transport properties accurate enough for engineering calculations.

In view of these considerations, an analytical study was conducted to determine and compare the relative heat-transfer coefficients and pressure drops of the inert gases and their binary mixtures. These inert gases include helium, neon, argon, krypton, and xenon. The viscosities and thermal conductivities of the binary mixtures necessary for the calculation of the relative heat-transfer coefficients and pressure drops were determined from the pure gas values by using available kinetic theory estimation techniques. The Prandtl number, a heat-transfer parameter of interest, was also determined for the binary mixtures. This report will present the mixture properties at example temperatures of 1000° and 2000° R and relative heat-transfer coefficients and pressure drops for both turbulent and laminar flow at an example temperature of 1000° R.

#### SYMBOLS

A	cross-sectional-flow area, sq ft
$C_p$	specific heat, Btu/(lb)(mass)(°R)
$D_H$	hydraulic diameter, ft
G	mass velocity, W/A, lb mass/(sq ft)(sec)
g	gravitational constant, 32.17 ft/sec <sup>2</sup>
H	heat-transfer factor
h	heat-transfer coefficient, Btu/(hr)(sq ft)(°R)
k	thermal conductivity, Btu/(hr)(ft)(°R)
L	length of conduit or duct, ft

M	molecular weight
m	molal flow, W/M, moles/sec
P	absolute pressure, lb/sq ft
$\Delta P$	pressure drop, lb/sq ft
Pr	Prandtl number
Re	Reynolds number
W	mass flow, lb mass/sec
X	mole fraction
$\mu$	viscosity, lb mass/(ft)(sec)
$\rho$	density, lb mass/cu ft
$\phi_{AB}$	coefficient for calculating viscosity and monatomic thermal conductivity
$\psi_{AB}$	coefficient for calculating monatomic thermal conductivity

#### Subscripts:

A,B	components of binary mixture
F	factor
L	laminar flow
M	mixture
T	turbulent flow
1,2	components of binary mixture

### METHOD OF ANALYSIS

Before determining the relative heat-transfer coefficients and pressure drops, the transport properties of these mixtures had to be determined. The viscosity and thermal conductivity of the binary mixtures were determined from the pure gas values by using available kinetic theory estimation techniques. The pure gas properties used in this analysis were obtained from reference 4 and are the latest available transport property data known to the author. These pure gas properties (presented in figs. 1 and 2) were obtained from equations based on rigorous kinetic theory using the Lennard-Jones (12-6) potential and are accurate enough for use in engineering calculations. The equations used to calculate the mixture transport properties were obtained from

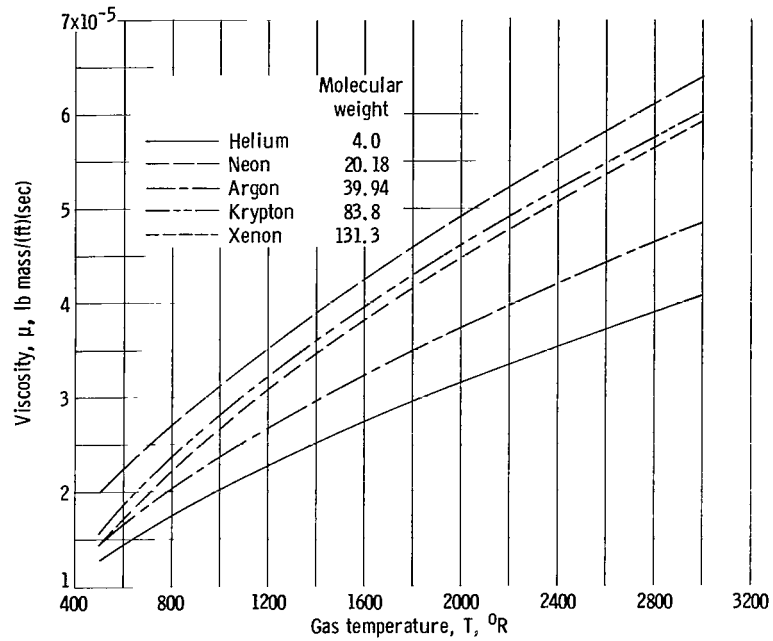


Figure 1. - Viscosities of inert gases.

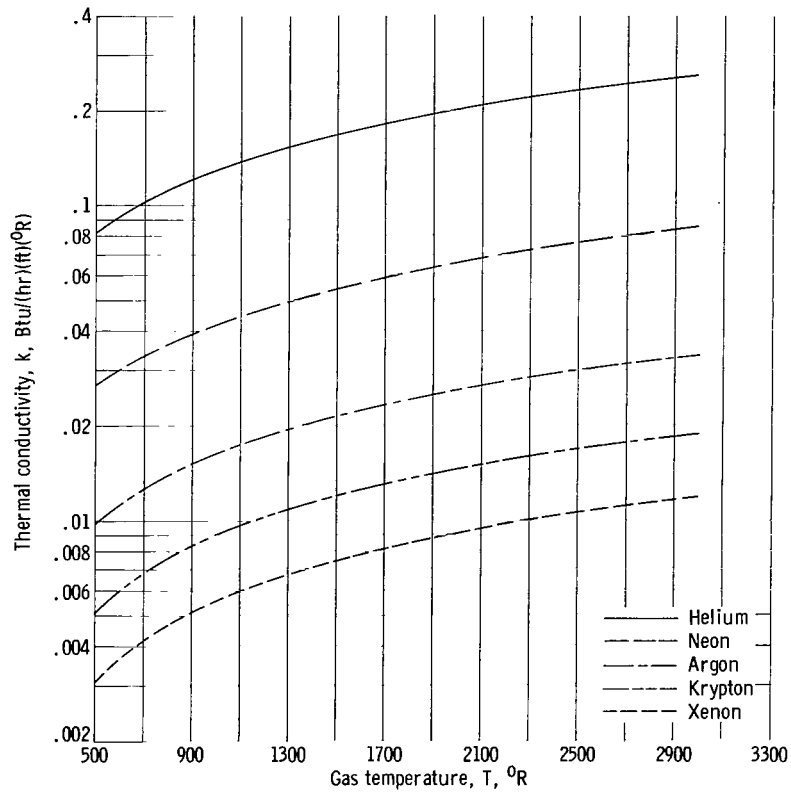


Figure 2. - Thermal conductivities of inert gases.

reference 5 and are presented in the appendix. These equations are approximate but yield transport properties that are accurate enough for use in engineering calculations. The Prandtl number, a heat-transfer parameter of interest, was also determined for these mixtures. With the properties determined, the relative heat-transfer coefficients and pressure drops can now be determined.

Relative heat-transfer coefficients and pressure drops are determined for both turbulent and laminar flow. These are relative values resulting from the assumption of a fixed heat-exchanger geometry and constant values of pressure, temperature, and molal flow rate. On this basis, the relative heat-transfer coefficients and pressure drops depend on only the transport properties, Prandtl number, and molecular weight of the fluid.

### Turbulent Flow

One of the commonly used equations to calculate the turbulent-flow heat-transfer coefficient for forced convection in a conduit or duct is (ref. 6)

$$h_T = 0.023 \frac{k}{D_H} \left( \frac{D_H G}{\mu} \right)^{0.8} Pr^{1/3} \quad (1)$$

where  $G = W/A = mM/A$ . If only those factors affecting the relative heat-transfer coefficient are considered, equation (11) shows that

$$h_T \propto \frac{kM^{0.8} Pr^{1/3}}{\mu^{0.8}} = H_T \quad (2)$$

where  $H_T$  is termed the heat-transfer factor for turbulent flow.

The pressure-drop equation for turbulent flow can be written as (ref. 6)

$$\frac{\Delta P}{P} = \frac{0.092 LG^2}{g D_H \rho P Re^{0.2}} \quad (3)$$

If only those factors affecting relative pressure drop are considered, equation (3) shows that

$$\frac{\Delta P}{P} \propto M^{0.8} \mu^{0.2} = \left( \frac{\Delta P}{P} \right)_{FT} \quad (4)$$

where  $(\Delta P/P)_{FT}$  is termed the pressure-drop factor for turbulent flow.

### Laminar Flow

Commonly used equations to calculate the laminar-flow heat-transfer coefficient (ref. 7) and pressure drop (ref. 7) for forced convection in a conduit or

duct are

$$h_L = 1.86 \frac{k}{D_H} \left( \frac{D_H G}{\mu} \right)^{1/3} Pr^{1/3} \left( \frac{D}{L} \right)^{1/2} \quad (5)$$

$$\frac{\Delta P}{P} = \frac{32 L G^2}{g D_H^5 Pr Re} \quad (6)$$

The equation used to calculate the heat-transfer coefficient is one of several suggested equations, which are approximately of the same form. Similar considerations for laminar flow as were made for turbulent flow yield

$$H_L = \frac{k M^{1/3} Pr^{1/3}}{\mu^{1/3}} \quad (7)$$

$$\left( \frac{\Delta P}{P} \right)_{FL} = \mu \quad (8)$$

Relative factors were then calculated by normalizing with respect to the lowest value for each comparison made. These relative factors yield valid comparisons of heat-transfer coefficients and pressure drops for a fixed heat-exchanger geometry and constant values for molal flow, pressure, and temperature.

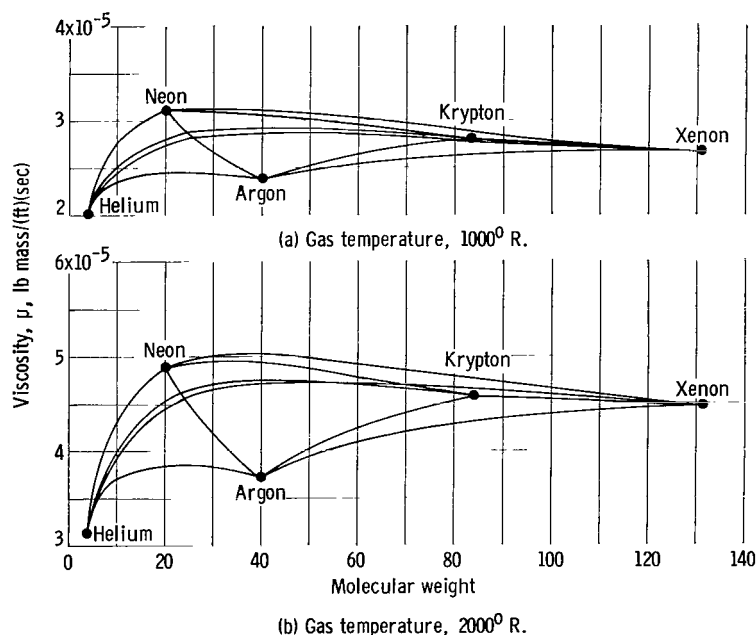


Figure 3. - Viscosities of binary mixtures of inert gases.



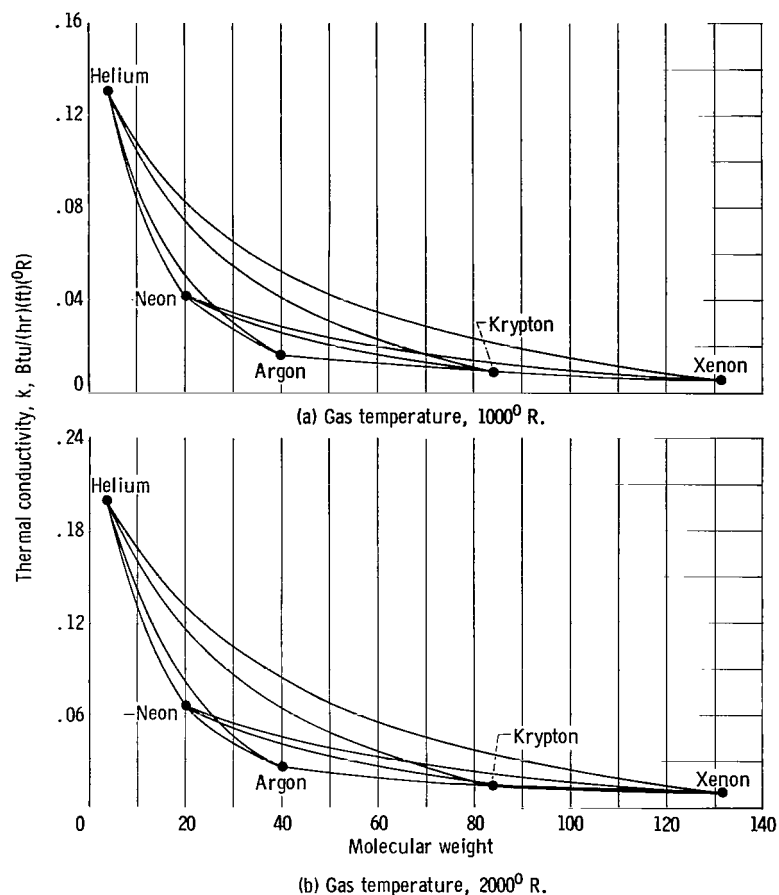


Figure 4. - Thermal conductivities of binary mixtures of inert gases.

## RESULTS OF ANALYSIS

The transport properties of the inert gases obtained from reference 4 are presented in figures 1 and 2 in engineering units instead of cgs units as in the reference. These properties are presented so that the reader can compute his own mixture properties at his conditions of interest with the presented equations. The mixture properties are discussed first and, then, the heat-transfer and pressure-drop comparisons are made.

### Mixture Properties

The viscosities, thermal conductivities, and Prandtl numbers of the binary mixtures are presented in figures 3, 4, and 5 for temperatures of 1000° and 2000° R. Two temperatures were selected in order to determine whether temperature has any effect on the property-value interrelationships (general shape of the cluster of curves) among the various mixtures for any given property.

Examination of these figures shows the following: (1) The basic relation among the various mixtures for any of the three properties is the same for the

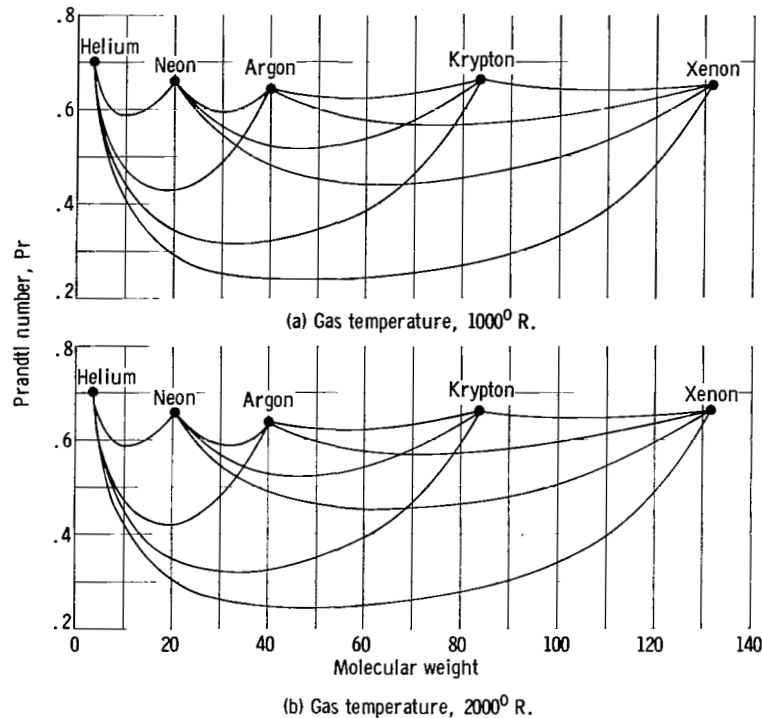


Figure 5. - Prandtl numbers of binary mixtures of inert gases.

selected temperatures of interest; (2) the mixture properties, in general, are not obtainable by common averaging methods using the pure gas values; and (3) for any given molecular weight, the properties of the mixtures differ significantly from each other as well as from those of the pure components. These figures also show that for a given molecular weight the thermal conductivities of the binary mixtures are higher and the Prandtl numbers lower than those of the pure components. The major differences in property values are seen to be caused by the helium in the binary mixtures containing helium.

#### Heat-Transfer and Pressure-Drop Comparisons

The heat-transfer and pressure-drop comparisons were initially made at two temperatures. Since the relative heat-transfer coefficients and pressure drops were almost the same for these two temperatures, the heat-transfer and pressure-drop comparisons will be presented at only one temperature. The nature of these comparisons at other temperatures will be similar except for some small difference in the magnitude of the effects. Since the relative heat-transfer coefficients and pressure drops are dependent on the fluid properties, the values for the binary mixtures, in general, cannot be obtained by common averaging methods using the pure gas values. The relative heat-transfer coefficients and pressure drops of the binary mixtures are presented for both turbulent and laminar flow at a temperature of 1000° R.

Turbulent flow. - The relative heat-transfer coefficients and pressure drops for turbulent flow are plotted against molecular weight in figures 6(a) and 6(b), respectively. As can be seen from figure 6(a), as the molecular

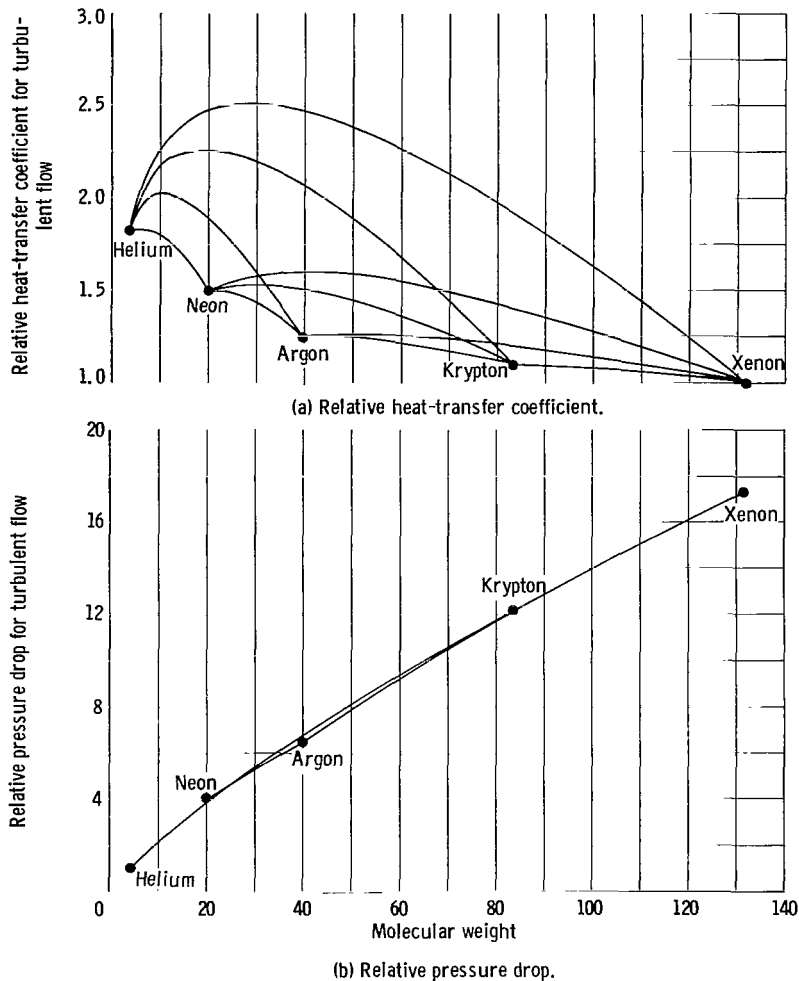


Figure 6. - Relative heat-transfer coefficients and pressure drops of inert gases and their binary mixtures for turbulent flow.

weight increases, the relative heat-transfer coefficient of the pure gases decreases and, for most mixtures, the relative heat-transfer coefficient increases to a maximum and then decreases to the value of the second component. The shape of these curves is primarily due to the decrease in the thermal conductivity and the increase in molecular weight as the molecular weight increases. Figure 6(a) also shows that for a given molecular weight, the heat-transfer coefficients of the binary mixtures are higher than those of the pure components. For example, at a molecular weight of 40, the heat-transfer coefficient for a helium-xenon mixture is approximately twice that of argon.

The relative pressure drop for turbulent flow is plotted against molecular weight in figure 6(b). As can be seen from figure 6(b), the relative pressure drop increases as the molecular weight increases, and the pressure drops of the binary mixtures are approximately equal to those of the pure gases. This behavior is caused by the pressure drop being primarily dependent on the molecular weight. Therefore, for a given molecular weight, the pressure drop for

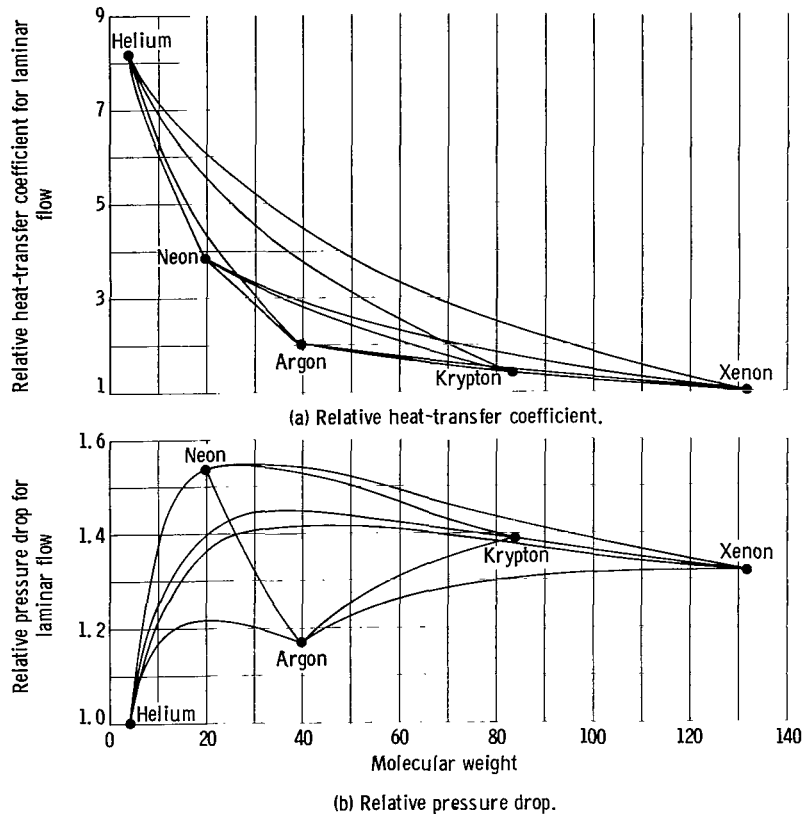


Figure 7. - Relative heat-transfer coefficients and pressure drops of inert gases and their binary mixtures for laminar flow.

turbulent flow remains approximately the same for all the binary mixtures, which allows a change in system working fluid without a significant change in pressure drop.

Laminar flow. - The relative heat-transfer coefficient for laminar flow is plotted against molecular weight in figure 7(a). As can be seen from figure 7(a), the relative heat-transfer coefficients of the pure components and the binary mixtures decrease as the molecular weight increases. This decrease in relative heat-transfer coefficient is caused by the decrease in the thermal conductivity. Figure 7(a) also shows that for a given molecular weight the laminar-flow heat-transfer coefficients of the binary mixtures are higher than those of inert gases. For example, at a molecular weight of 40, the heat-transfer coefficient of a helium-xenon mixture is approximately 2.25 times that of argon.

The relative pressure drop for laminar flow is plotted against molecular weight in figure 7(b). As can be seen from figure 7(b), the relative pressure drops of the binary mixtures for laminar flow behave similar to the viscosities of the binary mixtures. These behavior patterns are similar because the relative pressure drop is directly proportional to and a function of only viscosity. Therefore, for a given molecular weight, in laminar flow, a change of fluid (binary mixture) in the system results in a change in pressure drop as

well as change in heat-transfer coefficient, whereas in turbulent flow a change of fluid would not affect the pressure drop.

Application of results. - The heat-transfer and pressure-drop comparisons can be used to make first-order estimates of the effect of the working fluid on heat exchanger performance and/or size. These first-order estimates were based on the assumption that the gas is the controlling heat-transfer resistance in the heat exchanger. For example, it is desired to know what effect a change in working fluid from argon (molecular weight = 40) to a helium-xenon mixture of the same molecular weight would have on the performance of a given heat exchanger. For the case of turbulent flow, it is seen from figure 6 that the heat-transfer coefficient is doubled while the pressure drop remains the same. Therefore, the change in performance will be a doubling of the amount of heat transferred without a change in pressure drop. If a change in working fluid from argon to a helium-xenon mixture with a molecular weight of 75 is desired to reduce the number of turbomachinery stages, the heat-transfer coefficient and the pressure drop would increase approximately 70 percent.

Alternately, it may be desired to determine the effect of the working fluid on the required heat-exchanger size and pressure drop for maintaining a given heat-transfer performance. It can be determined from figure 6 that going from argon to a helium-xenon mixture of the same molecular weight results in a 50-percent decrease in both the length of the heat exchanger and the pressure drop. Since the heat-transfer coefficient is doubled, the same amount of heat can be transferred in only half the original length. The pressure drop per unit length remains constant and, therefore, pressure drop is reduced as length is reduced. For the change in working fluid from argon to a helium-xenon mixture with a molecular weight of 75, it can be shown from figure 6 that there would be about a 40-percent reduction in the size of the heat exchanger without any change in performance. This is due to the increase in both the heat-transfer coefficient and the pressure drop by a factor of 1.7. In order to obtain the same amount of heat transferred as with argon, the heat exchanger length can be reduced to  $1/1.7$  (about 60 percent) of its original value. This reduction in length would also result in the pressure drop being reduced to the same value as with argon.

These examples were for a fluid temperature of 1000° R, and it should be remembered that the magnitude of these effects are to some extent a function of temperature.

#### SUMMARY OF RESULTS

This analysis was conducted to determine and compare the relative heat-transfer coefficients and pressure drops of the inert gases and their binary mixtures. The required transport properties were determined from kinetic theory correlations, and the relative heat-transfer coefficients and pressure drops were determined from commonly used heat-transfer and pressure-drop equations by assuming a fixed heat-exchanger geometry and constant values of pressure, temperature, and molal flow rate. The pertinent results of this analysis are as follows:

1. The relative heat-transfer coefficients and pressure drops of the binary mixtures cannot be obtained by common averaging methods from the pure gas values. This is due to the behavior of the transport properties of the mixtures.

2. The effects of molecular weight on either the heat-transfer coefficients or the pressure drops are functions of the flow regime as well as the particular mixture. The relative heat-transfer coefficients of the pure gases decrease with an increase in molecular weight for both turbulent and laminar flow. As the molecular weight increases, the turbulent-flow heat-transfer coefficient for most mixtures increases to a maximum and then decreases, whereas the laminar-flow heat-transfer coefficient decreases. The pressure drop for turbulent flow increases with an increase in molecular weight while that for laminar flow shows no general trend.

3. For any given molecular weight, the heat-transfer coefficients of the binary mixtures are higher than those of a pure component for both turbulent and laminar flow. At a molecular weight of 40, for example, the turbulent-flow heat-transfer coefficient of a helium-xenon mixture would be approximately twice that of argon. For any given molecular weight, the turbulent-flow pressure drops for all mixtures are comparable to that of the pure component. The nature of the laminar-flow pressure-drop relation precludes any generalizations.

4. The substitution of a higher-molecular-weight binary gas mixture for a pure gas in a gas cycle system can result, in many cases, in more favorable turbomachinery and, at the same time, in improved performance from a given heat-transfer component or a smaller size for a required heat-transfer component. For the case of turbulent flow, as an example, the use of a helium-xenon mixture with a molecular weight of 75 instead of argon (molecular weight = 40) results in obtaining equal heat-transfer and pressure-drop performance from a heat exchanger that is reduced in size by 40 percent.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, November 25, 1964.

APPENDIX - METHOD OF CALCULATION OF TRANSPORT PROPERTIES  
OF BINARY MIXTURES OF INERT GASES

The viscosity of a mixture of gases is calculated by equation (13) of reference 5. For a binary mixture, the viscosity equation can be expressed as

$$\mu_M = \frac{\mu_1}{1 + \varphi_{12} \frac{X_2}{X_1}} + \frac{\mu_2}{1 + \varphi_{21} \frac{X_1}{X_2}} \quad (A1)$$

where

$$\varphi_{12} = \frac{\left[ 1 + \left( \frac{\mu_1}{\mu_2} \right)^{1/2} \left( \frac{M_2}{M_1} \right)^{1/4} \right]^2}{2\sqrt{2} \left( 1 + \frac{M_1}{M_2} \right)^{1/2}}$$

and

$$\varphi_{21} = \varphi_{12} \left( \frac{\mu_2}{\mu_1} \right) \left( \frac{M_1}{M_2} \right)$$

The thermal conductivity of a mixture of monatomic gases is calculated by equation (17) of reference 5. For a binary mixture, the thermal conductivity equation can be expressed as

$$k_M = \frac{k_1}{1 + \psi_{12} \frac{X_2}{X_1}} + \frac{k_2}{1 + \psi_{21} \frac{X_1}{X_2}} \quad (A2)$$

where

$$\psi_{AB} = \varphi_{AB} \left[ 1 + 2.41 \frac{(M_A - M_B)(M_A - 0.142M_B)}{(M_A + M_B)^2} \right] \quad \begin{array}{l} A = 1, B = 2 \\ \text{or} \\ A = 2, B = 1 \end{array}$$

and

$$\varphi_{AB} = \frac{\left[ 1 + \left( \frac{k_A}{k_B} \right)^{1/2} \left( \frac{M_A}{M_B} \right)^{1/4} \right]^2}{2\sqrt{2} \left( 1 + \frac{M_A}{M_B} \right)^{1/2}}$$

The component mole fractions  $X_1$  and  $X_2$  required for equations (A1) and (A2) are functions of the desired molecular weight of the mixture as well as those of the pure components. The molecular weight of a binary mixture can be expressed as

$$M_M = X_1 M_1 + X_2 M_2 \quad (A3)$$

Since a binary mixture is being considered,

$$X_1 = 1 - X_2 \quad (A4)$$

Substituting equation (A4) into equation (A3) and solving for  $X_2$  yield

$$X_2 = \frac{M_M - M_1}{M_2 - M_1} \quad (A5)$$

The Prandtl number is

$$Pr = 3600 \frac{C_P \mu}{k} \quad (A6)$$

where the 3600 is used to convert seconds to hours to make  $C_P \mu / k$  dimensionless. Since the specific heat of these gases and mixtures does not vary significantly with the temperatures and pressures of interest in the Brayton cycle systems, the ideal specific heat equation can be used. Therefore, for a monatomic gas

$$C_P = \frac{4.97}{M} \quad (A7)$$

Substituting equation (A7) into equation (A6) yields

$$Pr = \frac{17 \ 892 \mu}{M k} \quad (A8)$$

For a mixture of monatomic gases,

$$Pr = \frac{17 \ 892 \mu_M}{M_M k_M} \quad (A9)$$

The simple mixing rules presented are approximate, and the accuracy of the results depends on the relative sizes of the two components of the mixture. For large differences in molecular diameters, such as a mixture of helium and xenon, the results are accurate to within perhaps 10 to 15 percent, while for a small difference in molecular diameters, such as a mixture of argon and neon, the accuracy of the results is improved. A method that is based on rigorous kinetic theory is available, however, if design calculations are to be made for any given mixture; for this general parametric analysis, this complex method was not used.



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